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SERVICE NOTES

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A TECHNIQUE FOR CALIBRATING PHASE SHIFTERS TO HIGH ACCURACY

by

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INTRODUCTION

The general principle of the following method of calibrating phase shifters has been described in the literature [1-3], but the method is not well known and some of the details of the present version are believed to be novel. Because of the complexity of the apparatus and procedure, it is useful mainly for calibrating a primary standard of phase by means of which other phase shifters can be calibrated using a relatively simple comparison technique.

Section I gives the general theory of the method. Section II gives the details of the procedure used in calibrating Hewlett-Packard 885A series waveguide phase shifters for use as phase standards. Section III gives an analysis of the errors of the method and the precautions which should be taken to minimize them.

I. GENERAL THEORY

The transmission-type bridge circuit used in the calibration procedure is shown in Figure 1. A directional coupler splits the signal from the source into two parts, one of which passes through two phase shifters and the other through a single phase shifter and a variable attenuator. A second directional coupler brings the signals from the two branches together at the detector. Enough well-matched padding is used to make mismatch errors negligible. Simple tee junctions can be used as power dividers in place of the directional couplers. However, such junctions do not decouple the two branches of the bridge, so that more padding attenuation is necessary with a consequent loss in sensitivity of the bridge balance.

A null can be obtained at the detector by adjusting the variable attenuator in the lower branch and any one of the phase shifters. Because there are three phase shifters, there are many different combinations of settings of the phase shifters for which the bridge will be balanced. As will be shown, it is possible to exploit this multiplicity of balance conditions to calibrate any or all of the phase shifters in the bridge accurately knowing only the approximate calibrations of two of them to begin with.

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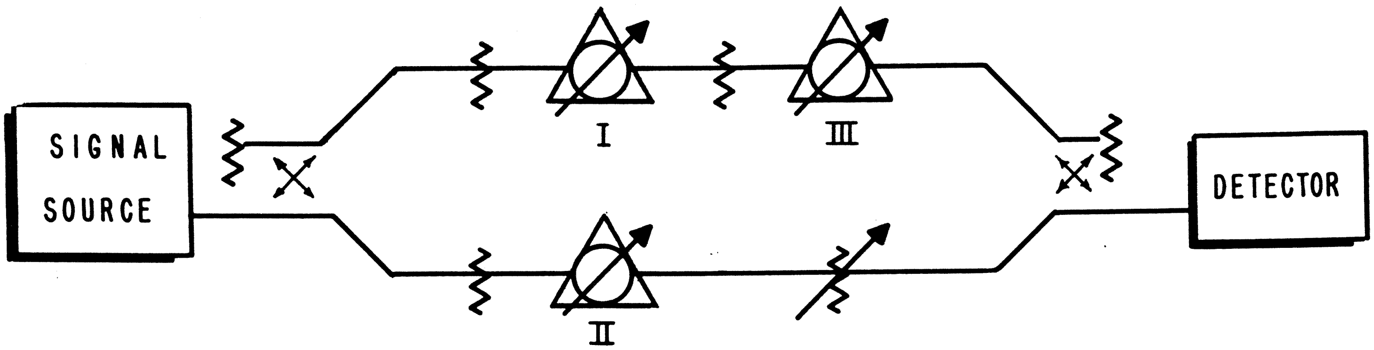


Figure 1.

The method depends on the possibility of measuring small phase changes with considerable accuracy by means of the corresponding changes in the detector output when the bridge is nearly, but not quite balanced. Two signals arriving at the detector through the different branches of the bridge are represented by vectors V_A and V_B in Figure 2.

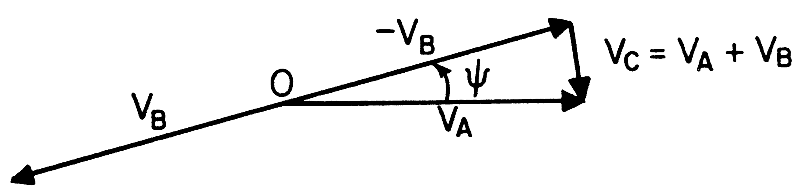


Figure 2.

The resultant signal at the detector is represented by $V_C = V_A + V_B$. Suppose that the bridge is initially balanced so that $V_C = 0$ and $V_A = -V_B$. If a phase shift is then introduced into one arm of the bridge, then the phase of $-V_B$ will be shifted relative to V_A, but its magnitude will remain the same. Hence the phase shift ψ can be determined from the signal level at the detector. If $|\psi| << 1$,

$$|\psi| \approx \frac{|V_C|}{|V_A|} \text{ radians} \qquad (1)$$

This method gives high absolute accuracy for small values of ψ. Furthermore, it can easily be seen that the result is insensitive to small incidental variations in $|V_B|$ relative to $|V_A|$. This point will be considered in more detail in the discussion of errors in Part III.

Let us assume that phase shifters I and II in Figure 1 are approximately calibrated. The following notation will be used:

θ = nominal phase shift

$\phi_I(\theta)$, $\phi_{II}(\theta)$ = actual phase shifts of I and II corresponding to nominal phase shift θ.

$\epsilon_I(\theta)$, $\epsilon_{II}(\theta)$ = errors in the nominal calibrations of I and II.

By definition we have the following relations:

$$\begin{aligned}\phi_I(\theta) &= \theta + \epsilon_I(\theta)\\ \phi_{II}(\theta) &= \theta + \epsilon_{II}(\theta)\end{aligned}$$

The following basic procedure, repeated for a sufficient number of values of the parameters, gives the data necessary for the calibration.

(1) Let phase shifters I and II be initially at settings θ_1 and θ_2 respectively. Adjust phase shifter III and the variable attenuator to obtain a null at the detector. Referring to Figure 2, this corresponds to $V_A = -V_B$, $V_C = 0$.

(2) Change the phase shift of III by about 180°, adjusting it so that the power at the detector is a maximum. This corresponds to $V_A = V_B$, $V_C = 2V_A$. Use this power level as a reference level.

(3) Change III again until the signal at the detector is some small fraction (accurately determined) of the reference level established in Step (2). The bridge is then nearly, but not quite, balanced, and we have $|V_C| = p\,|V_A|$ where $p \ll 1$. The choice of p is determined by the order of magnitude of the errors expected in the phase calibration. Typically, $p \doteq 0.1$.

Denoting the angle between V_A and $-V_B$ by ψ_o, we have

$$\psi_o \cong p \text{ radians}$$

In Figure 3 the vectors $\overrightarrow{OA_o}$ and $\overrightarrow{OB_o}$ represent the signals V_A and $-V_B$ after the completion of Step (3).

(4) Shift I and II through nominally the same angle θ_s, so that they read $\theta_1 + \theta_s$ and $\theta_2 + \theta_s$ respectively. The signals V_A and $-V_B$ are now represented by $\overrightarrow{OA}$ and $\overrightarrow{OB}$ in Figure 3. The new value of the phase angle between them is denoted by ψ. If there were no errors in the phase shifter calibrations, the two signals would be $\overrightarrow{OA'}$ and $\overrightarrow{OB'}$ with the same phase angle ψ_o between them as before. We have,

$$\begin{aligned}\angle AOA' &= \varphi_I(\theta_1 + \theta_s) - \varphi_I(\theta_1) - \theta_s = \epsilon_I(\theta_1 + \theta_s) - \epsilon_I(\theta_1)\\ \angle BOB' &= \varphi_{II}(\theta_2 + \theta_s) - \varphi_{II}(\theta_2) - \theta_s = \epsilon_{II}(\theta_2 + \theta_s) - \epsilon_{II}(\theta_2)\end{aligned}$$

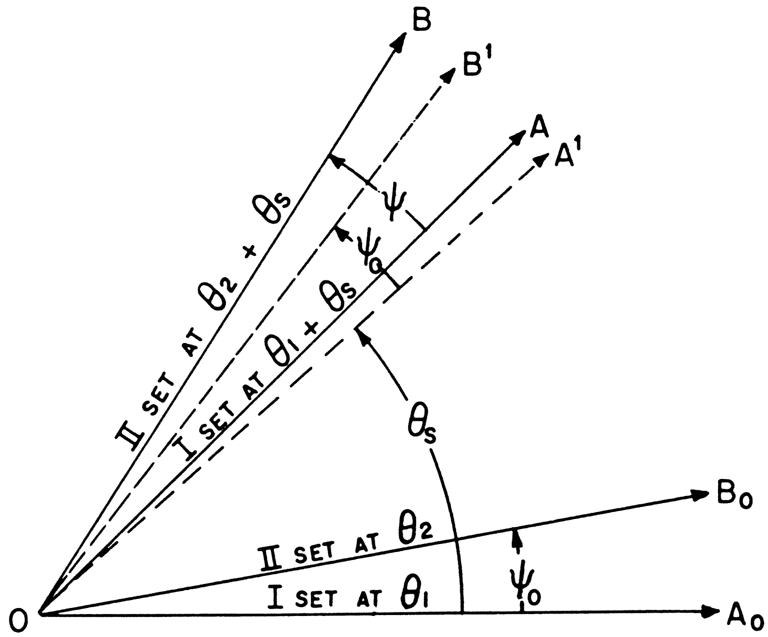


Figure 3.

Therefore,

$$\psi - \psi_o = \epsilon_I(\theta_1 + \theta_s) - \epsilon_I(\theta_1) - \left[\epsilon_{II}(\theta_1 + \theta_s) - \epsilon_{II}(\theta_1)\right]$$

$$\equiv \delta(\theta_1, \theta_2, \theta_s) \qquad (2)$$

The quantity δ can be found from the change in the RF signal at the detector produced by Step (4) relative to the reference level established in Step (3). Let D denote this change measured in decibels. We have,

$$\delta\,(\text{radians}) = p\,(10^{\frac{D}{20}} - 1) \qquad (3)$$

$$\cong 0.115\, p\, D \text{ if } \frac{D}{20} << 1$$

To calibrate the phase shifters we must measure the quantity $\delta(\theta_1, \theta_2, \theta_s)$ by the foregoing procedure for a number of values of θ_1, θ_2, θ_s which are whole multiples of some exact submultiple of 2π. There are many possible variations in the method. One possibility is to set $\theta_1 = 0$ and measure $\delta(0, \theta_2, \theta_s)$ where

$$\theta_2 = \frac{2\pi k}{N}, \quad k = 1, 2, \ldots N$$

$$\theta_s = \frac{2\pi n}{N}, \quad n = 1, 2, \ldots N$$

The correction of phase shifter I for $\theta = \frac{2\pi n}{N}$ can be found from the following relation:

$$\sum_{k=0}^{N-1} \delta\left(0, \frac{2\pi k}{N}, \frac{2\pi n}{N}\right) = N\left[\epsilon_I\left(\frac{2\pi n}{N}\right) - \epsilon_I(0)\right]$$

$$- \sum_{k=0}^{N-1}\left\{\epsilon_{II}\left[\frac{2\pi(n+k)}{N}\right] - \epsilon_{II}\left[\frac{2\pi k}{N}\right]\right\}$$

Solving for $\epsilon_I(\frac{2\pi n}{N})$ and canceling terms in the last summation gives:

$$\epsilon_I(\frac{2\pi n}{N}) = \epsilon_I(0) + \frac{1}{N}\sum_{k=0}^{N-1} \delta(0, \frac{2\pi k}{N}, \frac{2\pi n}{N}) + \frac{1}{N}\sum_{k=0}^{n-1} \left[\epsilon_{II}(2\pi + \frac{2\pi k}{N}) - \epsilon_{II}(\frac{2\pi k}{N})\right] \quad (4)$$

In the case of a microwave phase shifter, which is typically several wavelengths long, the choice of the zero setting is arbitrary. Therefore, we can set $\epsilon_I(0) = \epsilon_{II}(0) = 0$ without loss of generality.

The last term in Equation (4) will vanish if the phase shifter is of a type such that the settings θ and $\theta + 2\pi$ are physically identical. Even if this is not the case, however, the value of each of the terms in the summation can be found by a simple modification of the procedure for measuring $\delta(\theta_1, \theta_2, \theta_s)$. After performing Steps (1) - (3) of this procedure, shift II from nominal phase θ_2 to $\theta_2 + 2\pi$.

In the notation used before we now have,

$$\psi - \psi_o = \epsilon_{II}(\theta_2) - \epsilon_{II}(\theta_2 + 2\pi) \equiv \delta_{II}(\theta_2) \quad (5)$$

We can now rewrite Equation (4):

$$\epsilon_I(\frac{2\pi n}{N}) = \frac{1}{N}\sum_{k=0}^{N-1} \delta(0, \frac{2\pi k}{N}, \frac{2\pi n}{N}) - \frac{1}{N}\sum_{k=0}^{n-1} \delta_{II}(\frac{2\pi k}{N}) \quad (6)$$

The calibration corrections of II can now be found from the relation

$$\epsilon_{II}(\frac{2\pi n}{N}) = \epsilon_I(\frac{2\pi n}{N}) - \delta(0, 0, \frac{2\pi n}{N}) \quad (7)$$

Alternatively, the roles of I and II in the calibration process can be reversed, and the corrections of II found from the relation

$$\epsilon_{II}(\frac{2\pi n}{N}) = -\frac{1}{N}\sum_{k=0}^{N-1} \delta(\frac{2\pi k}{N}, 0, \frac{2\pi n}{N}) + \frac{1}{N}\sum_{k=0}^{n-1} \delta_I(\frac{2\pi k}{N}) \quad (8)$$

where $\delta_I(\theta_1) \equiv \epsilon_I(\theta_1 + 2\pi) - \epsilon_I(\theta_1)$ (9)

It is often advantageous in practice to take enough measurements so that ϵ_I and ϵ_{II} can be evaluated independently by means of Equations (6) and (8), after which Equation (7) can be used as a check.

If the phase shifter is to be calibrated at a large number of points, time can be saved by using a variation of this method consisting of two or more successively finer subdivisions. Let θ_a, θ_b be points at which phase shifters I and II have been calibrated by the first subdivision. Let it be required to calibrate the interval between θ_a and θ_b at M equally spaced points. The calibration corrections are given by the following equation, which can easily be verified:

$$\epsilon_I(\theta_a + m\Delta\theta) = \epsilon_I(\theta_a) + \frac{m}{M} \epsilon_I(\theta_b - \theta_a) + \frac{1}{M}\sum_{k=0}^{M-1} \delta(\theta_a, k\Delta\theta, m\Delta\theta) - \frac{1}{M}\sum_{k=0}^{m-1} \delta(0, k\Delta\theta, \theta_b - \theta_a) \tag{10}$$

$$\Delta\theta \equiv \frac{\theta_b - \theta_a}{M}$$

Since θ_a and θ_b are multiples of $\frac{2\pi}{N}$, the interval of the first subdivision, $\epsilon_I(\theta_a)$ and $\epsilon_I(\theta_b - \theta_a)$ are known quantities. For the special case where $\theta_a = 0$, $\theta_b = \frac{2\pi}{N}$, we have

$$\epsilon_I(\frac{2\pi m}{MN}) = \frac{m}{M}\epsilon_I(\frac{2\pi}{N}) + \frac{1}{M}\sum_{k=0}^{M-1} \delta(0, \frac{2\pi k}{MN}, \frac{2\pi m}{MN}) - \frac{1}{M}\sum_{k=0}^{m-1} \delta(0, \frac{2\pi k}{MN}, \frac{2\pi}{N}) \tag{11}$$

For $m = 1, 2, \ldots M$, Equation (11) gives the calibration corrections for $0 < \theta < \frac{2\pi}{N}$ at intervals of $\frac{2\pi}{MN}$. The rest of the range can be calibrated using other relations derived from Equation (10), but it is simpler to use the relation:

$$\epsilon_I(\frac{2\pi n}{N} + \frac{2\pi m}{MN}) = \epsilon_I(\frac{2\pi m}{MN}) + \epsilon_{II}(\frac{2\pi n}{N}) + \delta(\frac{2\pi m}{MN}, 0, \frac{2\pi n}{N}) \tag{12}$$

The expressions for ϵ_{II} corresponding to Equations (11) and (12) are as follows:

$$\epsilon_{II}(\frac{2\pi m}{MN}) = \frac{m}{M}\epsilon_{II}(\frac{2\pi}{N}) - \frac{1}{M}\sum_{k=0}^{M-1} \delta(\frac{2\pi k}{MN}, 0, \frac{2\pi m}{MN}) + \frac{1}{M}\sum_{k=0}^{m-1} \delta(\frac{2\pi k}{MN}, 0, \frac{2\pi}{N}) \tag{13}$$

$$\epsilon_{II}(\frac{2\pi n}{N} + \frac{2\pi m}{MN}) = \epsilon_{II}(\frac{2\pi m}{MN}) + \epsilon_I(\frac{2\pi n}{N}) - \delta(0, \frac{2\pi m}{MN}, \frac{2\pi n}{N}) \tag{14}$$

II. CALIBRATION OF HEWLETT-PACKARD 885A SERIES WAVE-GUIDE PHASE SHIFTERS FOR USE AS PHASE STANDARDS

A block diagram of a practical setup for calibrating waveguide phase shifters is given in Figure 4. The waveguide circuit is drawn approximately to scale, and the Hewlett-Packard model numbers of each of the components (for X-band) are given in parentheses.

The bridge circuit is basically that shown in Figure 1. A signal generator supplies 10 mw or more of R.F. power with 1000 C.P.S. amplitude modulation. The power level is monitored by means of a directional coupler, detector and voltmeter. A 3 db directional coupler is used as a power divider at each end of the bridge. In the lower branch of the bridge simple 10 db pads are used to isolate phase shifter I. In the upper branch two 10 db directional couplers are used as isolating attenuators around phase shifter II. The unused port of each coupler is terminated with a well matched load. The reasons for this difference between the lower and upper branches are given in the discussion of mismatch errors in Part III.

An uncalibrated variable attenuator is placed to the left of the auxiliary phase shifter (III) in the upper branch. The attenuator in the lower branch actually need not be accurately calibrated. However, it is desirable that it have the high resolution and small variation of phase shift for small attenuation values characteristic of the Hewlett-Packard 382A series attenuators.

Since 360° phase shift is produced by 180° rotation of the center section of a rotary waveguide phase shifter, there are two orientations of the center section corresponding to each dial reading. Since it is necessary to distinguish between these two orientations, the covers of phase shifters I and II should be removed so that the center sections can be directly observed.

A reference mark should be made to show the orientation of the center section corresponding to one 0° setting of each phase shifter.

The 885A series phase shifters have a friction clutch which permits the phase shift to be varied while the dial is clamped at a fixed reading. If it is desired to use one of the phase shifters as a reference standard after calibration, the clutch should be eliminated to prevent any possible error due to the clutch being shifted accidentally. This can be done by putting a dowel pin through the clutch sleeve into the shaft.

It has been found adequate to calibrate the 885A series phase shifters at intervals of 30°. The general scheme of the calibration procedure is first to determine the corrections to the dial calibrations at multiples of 90° and then to interpolate within each quadrant to find the corrections at multiples of 30°. In the notation of Part I this corresponds to $N = 4$, $M = 3$.

The detailed calibration procedure is given below. Typical data from an actual test run following this procedure are given in the Appendix.

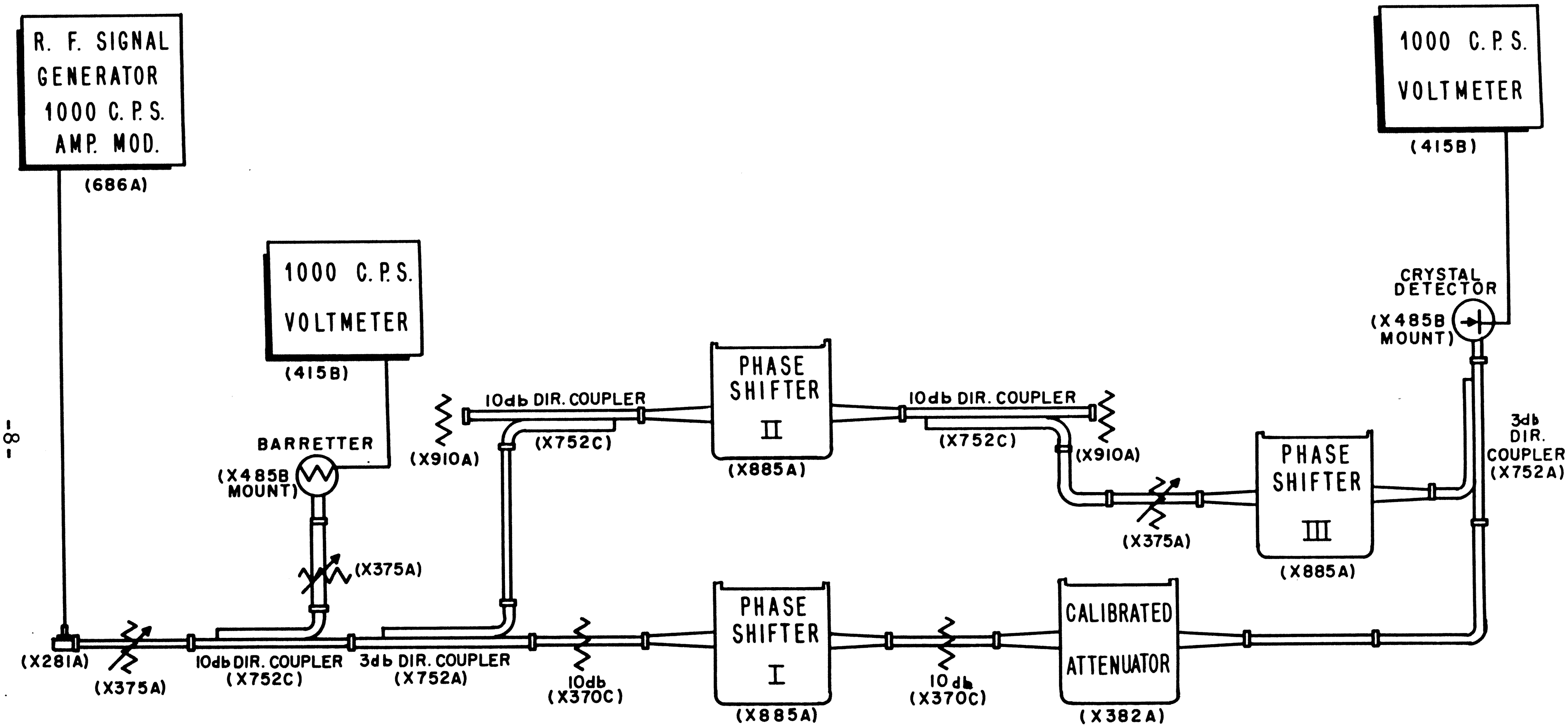


Figure 4. Apparatus for Calibrating X885A Phase Shifters

1. Measurement of Variation of Insertion Loss with Phase Settings of Phase Shifters II and III

Set the attenuator in the lower branch to maximum attenuation (> 70 db) so that a negligible amount of power reaches the detector through the lower branch of the bridge. Set the attenuator in the upper branch at about 5 db. Adjust the power from the source and the input attenuator so that the detector is operating in the square law region, but well above the noise level of the voltmeter. Set phase shifter II to 0^o, and adjust the voltmeter reading to some convenient reference level by means of the gain control. Turn II through two complete cycles of phase shift (corresponding to one complete rotation of the center section), and record the variation of the R. F. level at the detector at multiples of 30^o.

Repeat this procedure for phase shifter III.

The insertion loss of II at phase setting θ will be denoted by $A_{II}(\theta)$, and that of III, by $A_{III}(\theta)$.

2. Balancing the Bridge

Adjust the input attenuator so that the R. F. level at the detector is at least 40 db above the minimum detectable signal. Set the attenuator in the lower branch at 0 db. Set phase shifters I and II at 0^o, noting the orientation of the center sections. Adjust phase shifter III and the attenuator in the lower branch alternately until a null is obtained at the detector. The component of the signal at the detector reaching it through the upper branch of the bridge is now equal in magnitude and opposite in phase to the component reaching it through the lower branch. In the balance condition the attenuator in the lower branch should be in the range from 2 to 8 db. Record the dial reading of III in the balance condition.

3. Setting a Reference Level with Signal Components in Phase

Turn phase shifter III through approximately 180^o from the balance condition, adjusting it to maximize the detector output. The two components of the signal at the detector are now in phase and still equal in magnitude. Adjust the input attenuator so that the power level at the detector is at a convenient reference level, low enough so that the detector is operating in the square law region, but well above noise.

4. Setting the Bridge to 5.73^o of Phase Unbalance

Adjust the input attenuator to increase the power level entering the bridge by exactly 26 db. This can be measured by means of the detector used to monitor power, either by relying on the square-law characteristics of a barretter or by using a calibrated attenuator in front of the detector.

Adjust phase shifter III so that the detector output is returned to the reference level established in Step 3. There are two settings of III satisfying this condition. Choose the setting for which a slight increase of the phase reading on the outer scale of numerals decreases the detector output. This choice will result in the corrections having the right signs if the measurements are made as described in the following steps. Record this phase setting of III, which will be denoted by θ_o.

The bridge now has a small and accurately known degree of phase unbalance. The component signals at the detector are related as shown in Figure 2. The resultant voltage V_C has a magnitude one-tenth that of the components V_A and V_B, so that the phase difference between these components is 0.1 radian = 5.73°.

5. Measurement of D (0, 0, 30°).

Shift phase shifters I and II so that they both read 30° (on the outer, clockwise-increasing scale of numerals). Use the inner dial with marks every 0.5° to make the setting accurately. Adjust the attenuator in the lower branch to minimize the detector output. Any change in the setting which is necessary to accomplish this is due to small variations of the insertion loss of phase shifters I and II with phase setting. The necessary adjustment of the attenuator should not exceed 0.5 db, and is typically much less than this. Over this range the phase shift of a rotary-vane type attenuator like the X382A is negligible.

Record the change in power level relative to the reference level established in Step 4. This change will be denoted by D (0, 0, 30°), following the notation of Part I. An increase in the power level relative to the reference level (i.e. a decrease in the reading on the decibel scale) should be recorded as a positive value.

6. Measurement of D (0, 0, 30n°)

Repeat Step 5 for settings of phase shifters I and II equal to all multiples of 30° up to 360°. The change in the power level after each of these steps (relative to the reference level established in Step 4) is denoted by D (0, 0, 30n°) (n = 2, 3, ... 12).

7. Measurement of D (0, 30°, 30n°)

Set phase shifter I to 0° and phase shifter II to 30°. Adjust III to return the voltmeter reading to the reference level established in Step 4. Adjust the attenuator in the lower branch for a minimum, and, if any change is necessary, readjust phase shifter III to return to the reference level.

Turn both I and II through 30° (nominal) phase shift, so that I reads 30° and II reads 60°. Adjust the attenuator in the lower branch to obtain a minimum. Record the change in power level at the detector, which is denoted by D (0, 30°, 30°).

Turn I and II simultaneously through multiples of 30° so as to measure D (0, 30°, θ_s) for θ_s = 60°, 90°, 180°, 270°.

8. Measurement of Other Values of D $(0, \theta_2, \theta_s)$

Repeat Step 7 with different initial settings of phase shifter II so as to measure D $(0, \theta_2, \theta_s)$.

(a) $\theta_2 = 60^\circ, 90^\circ; \theta_s = 30^\circ, 60^\circ, 90^\circ, 180^\circ, 270^\circ$.
(b) $\theta_2 = 180^\circ, 270^\circ; \theta_s = 90^\circ, 180^\circ, 270^\circ$.

9. Measurement of D $(\theta_1, 0, \theta_s)$

Repeat Steps 7 and 8 with the roles of phase shifters I and II reversed, so as to measure D $(\theta_1, 0, \theta_s)$ for the same set of values as before with θ_2 replaced by θ_1. (This step is only necessary to provide a check of the data and may be omitted if desired.)

10. Measurement of $D_I(0)$

Set I and II to 0° again. Adjust III to return to the reference level established in Step 4. Turn I through 360°. Record the change in the level, which will be denoted by $D_I(0)$. This quantity is different from zero in the 885A series phase shifters only because of slight asymmetries resulting from the finite tolerances on the dimensions of the parts which can be held in production.

11. Measurement of $D_I(\theta)$

Set I to 30° (in the first half-cycle of rotation of the center section); set III to return to the reference level; and turn phase shifter I through 360° to measure $D_I(30^\circ)$. Similarly measure $D_I(\theta)$ for all multiples of 30° up to 360°.

12. Measurement of $D_{II}(\theta)$

Repeat Steps 10 and 11 shifting II instead of I so as to obtain the corresponding quantity for II, denoted by $D_{II}(\theta)$, for all multiples of 30°.

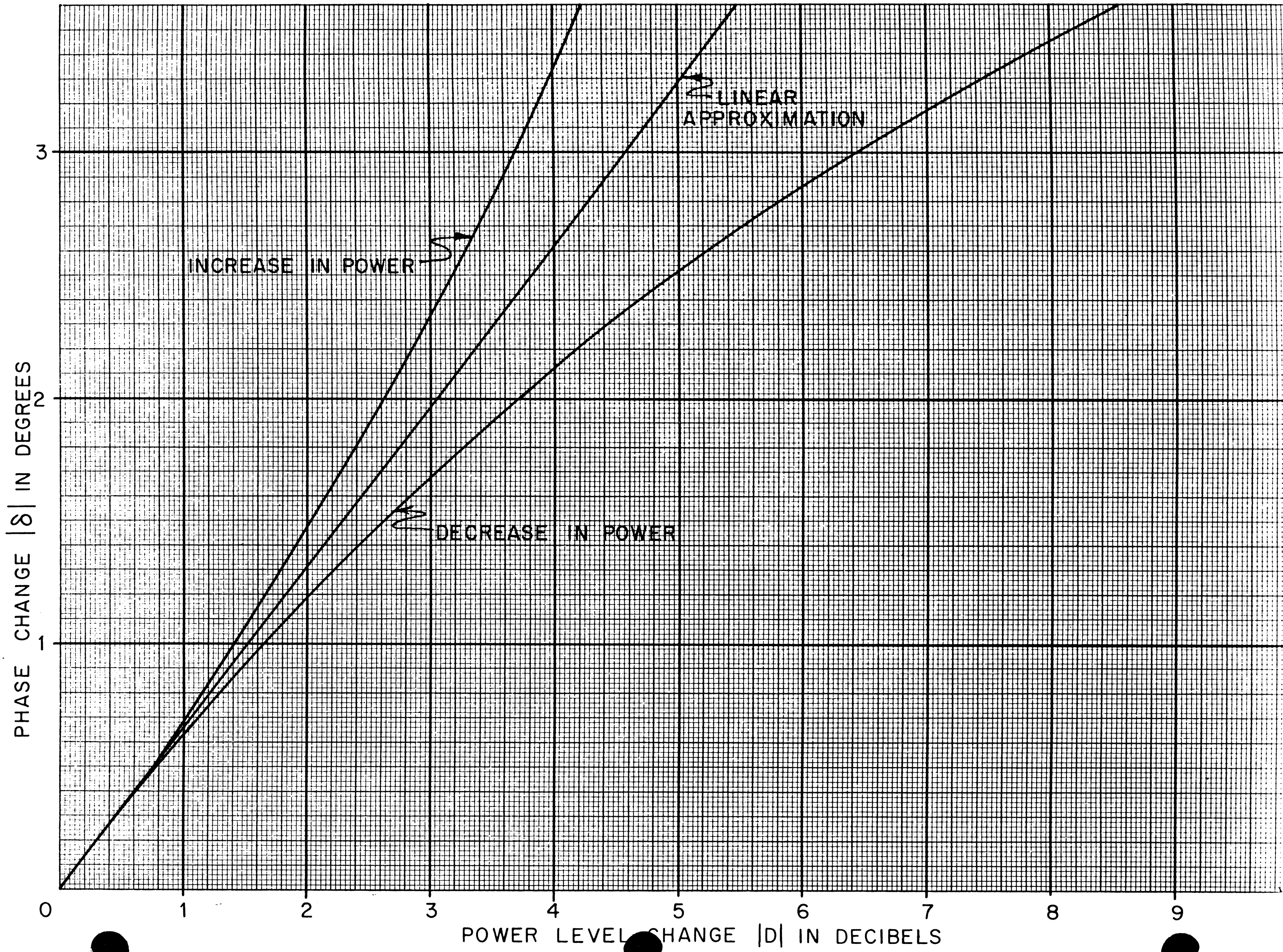
It should be noticed that the sign conventions adopted in Part I (Equations (5) and (9)) in the definitions of $\delta_{II}(\theta)$ and $\delta_I(\theta)$ are consistent with the fact that phase shifts of the same sign in the upper and lower branches of the bridge produce changes of opposite sign in the resultant signal at the detector.

The reduction of the data to find the corrections to the phase shifter dial readings is carried out in the following steps:

(1) Correction for the Variation of Insertion Loss of Phase Shifters II and III

The measured values of D can be corrected for the error resulting from the variation of insertion loss of II and III by means of the following relations, where D' denotes the corrected value:

Figure 5



LINEAR
APPROXIMATION
INCREASE IN POWER
DECREASE IN POWER
PHASE CHANGE |δ| IN DEGREES
POWER LEVEL CHANGE |D| IN DECIBELS
0
1
2
3
4
5
6
7
8
9

$$D'(0, \theta_2, \theta_s) = D(0, \theta_2, \theta_s) + A_{II}(\theta_2 + \theta_s) - A_{II}(\theta_2)$$
$$- 0.115 \left[A_{III}(\theta_o - \theta_2) - A_{III}(\theta_o + 186^o)\right] D(0, \theta_2, \theta_s) \quad (15)$$

$$D'(\theta_1, 0, \theta_s) = D(\theta_1, 0, \theta_s) + A_{II}(\theta_s) \quad (16)$$
$$- 0.115 \left[A_{III}(\theta_o + \theta_1) - A_{III}(\theta_o + 186^o)\right] D(\theta_1, 0, \theta_s)$$

The terms involving A_{III} are nearly always negligible. The corrections for D_I and D_{II} may be found similarly but should be negligible.

(2) Calculation of δ from D'

Using the corrected value D' of D, setting p = 0.1, and expressing δ in degrees, Equation (3) becomes

$$\delta \text{ (degrees)} = 5.73 \left(10^{\frac{D'}{20}} - 1\right) \quad (17)$$
$$\cong 0.660D' \text{ if } \frac{D'}{20} << 1$$

In practice the linear approximation for δ in terms of D' is not always quite good enough. Figure 5 shows a plot of Equation (17), from which δ can be found for any given value of D'. δ_I, δ_{II} can be found similarly from D_I, D_{II}.

(3) Calculation of the Calibration Corrections

The calibration corrections ϵ_I, ϵ_{II} can be found from Equations (6) - (14) in Part I. Setting N = 4 and M = 3, we have

$$\epsilon_I(90n) = 1/4 \sum_{k=0}^{3} \delta(0, 90k, 90n) - 1/4 \sum_{k=0}^{n-1} \delta_{II}(90k), \quad (18)$$
$$n = 1, 2, 3$$

$$\epsilon_I(30m) = \frac{m}{3} \epsilon_I(90) + 1/3 \sum_{k=0}^{2} \delta(0, 30k, 30m)$$
$$-1/3 \sum_{k=0}^{m-1} \delta(0, 30k, 90), \quad (19)$$
$$m = 1, 2$$

$$\epsilon_I(90n + 30m) = \epsilon_I(30m) + \epsilon_{II}(90n) + \delta(30m, 0, 90n), \quad (20)$$
$$n = 1, 2, 3; m = 1, 2$$

$$\epsilon_{II}(90n) = -1/4 \sum_{k=0}^{3} \delta(90k, 0, 90n) + 1/4 \sum_{k=0}^{n-1} \delta_I(90k), \quad (21)$$
$$n = 1, 2, 3$$

$$\epsilon_{II}(30m) = \frac{m}{3}\,\epsilon_I(90) - 1/3 \sum_{k=0}^{2} \delta(30k,\ 0,\ 30m)$$

$$+ 1/3 \sum_{k=0}^{m-1} \delta(30k,\ 0,\ 90), \qquad m = 1,\ 2 \tag{22}$$

$$\epsilon_{II}(90n + 30m) = \epsilon_{II}(30m) + \epsilon_I(90n) - \delta(0,\ 30m,\ 90n), \qquad n = 1,\ 2,\ 3;\ m = 1,\ 2 \tag{23}$$

When the setting of an 885A series phase shifter is changed from 0 to θ, using the outer scale with clockwise-increasing numerals, the change in phase shift is approximately $-\theta$. Denoting the absolute value of the actual change in phase shift by $\phi(\theta)$ as in Part I, we have

$$\phi_I(\theta) = \theta + \epsilon_I(\theta)$$

and

$$\phi_{II}(\theta) = \theta + \epsilon_{II}(\theta)$$

for phase shifters I and II respectively.

III. ANALYSIS OF ERRORS

The following are believed to be the most important sources of error in the calibration procedure described in Part II.

1. Errors in the Measurement of the 26 db Change in Power Level

(Step 4, page 9).

This change can be measured with an accuracy of ±0.2 db with ordinary precautions. The corresponding error in the quantity D increases as the magnitude of D increases. Assume that the maximum positive and negative values of D are those corresponding to $\delta = \pm 3.6^{o}$. (This limit should never be exceeded with 885A phase shifters.) From Figure 5 we find that the maximum error is δ for either a positive or a negative value of D is $.08^{o}$.

From Equations (18) and (21) it can be seen that the error in ϵ (90n) (n=1, 2, 3) should not exceed that in any of the values of δ from which it is computed. (The errors in the quantities δ_I, δ_{II} should be negligible, because these quantities never exceed a few tenths of a degree in magnitude.) From Equations (20) and (23) we see that the error in those values of ϵ (θ) which are obtained by interpolation within each quadrant may be as much as three times the error in any of the values of δ from which they are computed. Hence in this case the maximum error in ϵ due to the uncertainty in the measurement of the 26 db change in power level is $.24^{o}$.

2. Errors in the Measurement of D Due to Detector Law and Voltmeter Errors

In Steps (5)-(9) of the calibration procedure, it is necessary to measure variations of the RF level which may amount to several decibels. Detector law and voltmeter errors in the measurement of this variation should be less than 0.1 db in a value of D, or $.07^{o}$ in the corresponding value of δ. As before, the maximum error in a value of ϵ due to this source will be $3 \times .07^{o} = .21^{o}$.

3. Mismatch Errors

Because of the relatively poor match of the 885A series phase shifters, special precautions must be taken to ensure that the line on both sides of the phase shifter under test is well matched. For the greatest accuracy it is necessary to put a slide screw tuner into the line on each side of the phase shifter and tune out the mismatches, replacing the phase shifter by a high-directivity directional coupler to monitor the reflected power during the tuning process. This is a cumbersome and time consuming procedure, however, especially when calibrations are to be made at a number of different frequencies. If some sacrifice in accuracy can be tolerated, a sufficiently good match can be obtained by using multihole directional couplers as attenuator pads, as shown in Figure 3.

An upper limit for the mismatch of the line on each side of phase shifter II in Figure 3 can be found from the published specifications of the 752C couplers, 914A loads and other components. However, if the actual mismatches in the assembled bridge are measured at each of frequencies where the phase shifter is to be calibrated, they are usually found to be much lower than the calculated upper limits. These measured values of the mismatches can then be used in calculating the maximum phase error from this source.

In an actual bridge similar to that shown in Figure 3 the VSWR of the line on either side of phase shifter II was found to be less than 1.04 at any of the frequencies at which the phase calibration was to be made. Using this figure and the upper limit of 1.35 specified for the VSWR of the 885A, the maximum error in the phase calibration due to the mismatch of phase shifter II is found to be 0.37°.

Usually the phase shifter calibrated by the primary calibration technique is to be used as a standard against which other phase shifters are calibrated by comparison. If the same bridge is used in the comparison procedure as was used in the primary calibration, the effect of the mismatch of the line on either side of the phase shifter used as standard is the same during the comparison procedure as during the primary calibration. Therefore, these mismatches do not introduce any error in the calibration of the phase shifter which is being compared with the standard. The bridge shown in Figure 3 was designed with the intention of using phase shifter I as a standard, the phase shifter under test being placed in the position of II. Consequently, the two 10 db attenuator pads in the lower branch of the bridge need not be especially well matched,

4. Random Errors

Under this heading will be included errors due to short term instability of the R.F. power level or of any components in the system and to the finite resetability of the phase shifters. These errors are best evaluated by repeating a set of phase calibration measurements several times to find the degree of reproducibility of the results. Analysis of a series of runs of this kind has shown that a value of D is reproducible with a standard error of about .07 db, corresponding to an error of $.05^{\circ}$ in δ. This gives a standard error in ϵ (in the worst case) of $\sqrt{3} \times .05^{\circ} = .09^{\circ}$. The maximum random error in a value of ϵ may be estimated as three times the standard error, or $.27^{\circ}$.

5. Errors Due to the Variation of Insertion Loss of the Phase Shifters with Phase Setting

Figure 6 shows the components of the signal at the detector after completing the procedure for measuring a value of D (e.g. after Step 5 on page 10).

$\overrightarrow{0A}$ represents the component reaching the detector through the lower branch of the bridge, and $\overrightarrow{0B}$, the negative of the component reaching it through the upper branch. $\overrightarrow{0A_0}$ and $\overrightarrow{0B_0}$ represent the values these components would have if there were no variation of insertion loss with phase. 0A' is the lower-branch component before the final operation of tuning for minimum output with the attenuator in the lower branch.

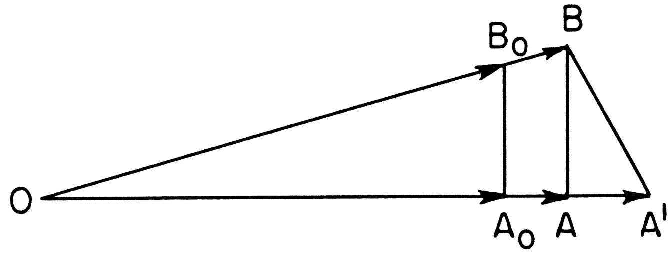


Figure 6

Assume $\overline{0B_o} = 1$ (since it corresponds to the reference level), and adopt the following notation:

$\overline{0A} - \overline{0A_o} = q_1$ = change in amplitude of lower-branch signal

$\overline{0B} - 1 = q_2$ = change in amplitude of upper-branch signal

$\overline{AB} = c$

$\overline{A'B} = c'$

The following relations hold:

$$c = (1 + q_2) \sin \gamma \qquad (24)$$

$$c' = \sqrt{c^2 + (q_1 - q_2 \cos \gamma)^2} = \sin \gamma \sqrt{(1 + q_2)^2 + \left(\frac{q_1 - q_2 \cos \gamma}{\sin \gamma}\right)^2} \qquad (25)$$

Equation (24) shows that the effect of the variation of insertion loss of phase shifter II can be compensated for simply by correcting the reference level by an amount equal to the change in insertion loss of II.

The effect of the variation of insertion loss of phase shifter III is equivalent to that of an error in measuring the 26 db change in level in Step 4, page 9. A given variation of insertion loss of III produces a much smaller error in D than an equal variation in II, and the effect of the variation of III is usually negligible. However, it can be corrected for if necessary.

These corrections are given in Equations (15) and (16). Since they amount to only a few tenths of a decibel at most, the error in determining these correction terms should be negligible.

If the operation of tuning the attenuator in the lower branch for minimum output is omitted, it is still possible in principle to correct the data for variation of insertion loss by means of Equation (25). However, this is much more complicated than Equation (24), and, furthermore, the corrections may be several times larger than in the first case, making them harder to determine accurately.

The table below summarizes the errors discussed above. In addition to the errors in the primary phase calibration, the errors introduced in comparing another phase shifter with the standard are given. The columns labeled A are computed for values of ϵ obtained by two successive subdivisions of 360° (for example, $\epsilon(\theta)$ for $\theta = 30°, 60°, 120°$, etc. in the case discussed in Part II).

Source of Error	Max. Error in Primary Calib.		Max. Error in Comparison with Standard	
	A	B	A	B
1. Errors in measuring 26 db change in level	.24°	.08°	.08°	.08°
2. Detector and meter errors in measuring D_1	.21	.07	.07	.07
3. Mismatch errors	-	-	.37	.09
4. Random errors	.27	.15	.15	.15
Total Error	.72°	.30°	.67°	.39°

Total error, standard + comparison:

A. .72° + .67° = 1.39°

B. .30° + .39° = .69°

The columns labeled B are computed for values of ϵ obtained by a single subdivision of 360° (for example, $\epsilon(\theta)$ for $\theta = 90°, 180°, 270°$ in the case discussed in Part II). Furthermore, in case A it is assumed that the line into which the phase shifter under test is inserted has a maximum VSWR in either direction of 1.04, whereas in case B the maximum VSWR is assumed to be 1.01. Consequently, case A gives the accuracy attainable when some time-saving simplifications are made in the procedure, whereas case B gives the accuracy attainable when more measurements are made and more elaborate precautions are taken.

REFERENCES:

(1) P. G. Smith, M. I. T. Master's Thesis, 1948.

(2) R. M. Brown & A. J. Simmons, "Dielectric Quarter-wave and Half-wave Plates in Circular Waveguide" N. R. L. Report #4218, Appendix B.

(3) D. A. Alsberg and D. Leed, "A Precision Direct-Reading Phase and Transmission Measuring System for Video Frequencies" B. S. T. J., April, 1949, p. 233.

<u>APPENDIX</u>

Calibration Data for Standard X885A's

Date: 9/4/57

X885A Serial # S1 is Position I
X885A Serial # S2 is Position II

Measurements by: Barnett

Frequency: 12.4 kmc

1. Variation of Insertion Loss with Phase Setting

$\theta \longrightarrow$

	30	60	90	120	150	180	210	240	270	300	330	360
$A_{II}(\theta)$	-.06	-.05	.01	.08	.11	.03	-.05	-.03	.05	.12	.10	.00
$A_{II}(\theta+360)$	Same as $A_{II}(\theta)$											
$A_{III}(\theta)$	-.02	-.01	.00	.00	.05	.11	.13	.09	.04	-.01	-.03	.00
$A_{III}(\theta+360)$	Same as $A_{III}(\theta)$											

2. Phase Calibration Data for I: $D = D(0, \theta_2, \theta_s)$ in db

$d = A_{II}(\theta_2+\theta_s)-A_{II}(\theta_2)$, $D^1 = D+d$

$\theta_s \longrightarrow$

$\theta_2 \downarrow$		0	30	60	90	120	150	180	210	240	270	300	330
0	D	0	.65	1.10	.95	.65	.85	1.25	1.95	2.05	1.45	.55	-.35
	d		-.06	-.05	.01	.08	.11	.03	-.05	-.03	.05	.12	.10
	D'		.59	1.05	.96	.73	.96	1.28	1.90	2.02	1.50	.67	-.25
30	D	0	.95	1.05	.55			1.65			1.30		
	d		.01	.07	.14			.01			.18		
	D'		.96	1.12	.69			1.66			1.48		
60	D	0	.55	.20	-.45			1.75			.15		
	d		.06	.13	.16			.02			.15		
	D'		.61	.33	-.29			1.77			.30		
90	D	0			-1.00			1.90			-.25		
	d				.02			.04			-.01		
	D'				-.98			1.94			-.26		
180	D	0			1.60			2.00			1.80		
	d				.02			-.03			-.02		
	D'				1.62			1.97			1.78		
270	D	0			-1.00			1.45			-.90		
	d				-.05			-.04			-.02		
	D'				-1.05			1.41			-.92		
$D_{II}(\theta)$		-.05	-.10	.00	.00	.05	.10	.20	.10	-.15	-.20	-.10	.00

3. Phase Calibration data for II: $D = D(\theta_1, 0, \theta_s)$ in db

$d = A_{II}(\theta_s)$, $D' = D + d$

$\theta_s \longrightarrow$

$\theta_1 \downarrow$		0	30	60	90	120	150	180	210	240	270	300	330
0	D	0	.65	1.10	.95	.65	.85	1.25	1.95	2.05	1.45	.55	-.35
	d		-.06	-.05	.01	.08	.11	.03	-.05	-.03	.05	.12	.10
	D'		.59	1.05	.96	.73	.96	1.28	1.90	2.02	1.50	.67	-.25
30	D	0	.10	.40	.45			1.15			.20		
	d		-.06	-.05	.01			.03			.05		
	D'		.04	.35	.46			1.18			.25		
60	D	0	-.05	.45	1.15			.60			.05		
	d		-.06	-.05	.01			.03			.05		
	D'		-.11	.40	1.16			.63			.10		
90	D	0			2.00			-.15			.70		
	d				.01			.03			.05		
	D'				2.01			-.12			.75		
180	D	0			-.55			-2.80			-.35		
	d				.01			.03			.05		
	D'				-.54			-2.77			-.30		
270	D	0			.05			-.55			1.90		
	d				.01			.03			.05		
	D'				.06			-.52			1.95		
$D_I(\theta)$		-.05	.00	.00	.15	.10	.05	-.25	-.15	.00	.10	.20	.10

4. Reduction of data for I: $\delta = \delta(0, \theta_2, \theta_s)$ in degrees

(a) δ computed directly from D (Section 1)
(b) δ computed from ϵ_I, ϵ_{II} (Section 6)
(c) Difference (a)-(b)

$\theta_s \longrightarrow$; $\theta_2 \downarrow$

θ_2		0	30	60	90	120	150	180	210	240	270	300	330
0	(a)	0	.40	.74	.66	.49	.66	.90	1.40	1.49	1.08	.45	-.17
	(b)		.37	.71	.62	.53	.72	.92	1.48	1.51	1.11	.49	-.13
	(c)		.03	.03	.04	-.04	-.06	-.02	-.08	-.02	-.03	-.04	-.04
30	(a)	0	.66	.79	.47			1.20			1.06		
	(b)		.67	.73									
	(c)		-.01	.06									
60	(a)	0	.41	.22	-.19			1.29			.20		
	(b)		.39	.28									
	(c)		.02	-.06									
90	(a)	0			-.62			1.42			-.17		
	(b)				-.60			1.46			-.12		
	(c)				-.02			-.04			-.05		
180	(a)	0			1.17			1.45			1.30		
	(b)				1.16			1.45			1.13		
	(c)				.01			.00			.17		
270	(a)	0			-.66			1.00			-.58		
	(b)				-.61			.94			-.50		
	(c)				-.05			.06			-.08		
$\delta_{II}(\theta_s)$		-.03	-.07	.00	.00	.03	.07	.13	-.07	-.10	-.13	-.07	.00

5. Reduction of data for II: $\delta = \delta(\theta_1, 0, \theta_s)$ in degrees

(a) δ computed directly from D (Section 2)

(b) δ computed from ϵ_I, ϵ_{II} (Section 6)

(c) Difference (a)-(b)

$\theta_1 \downarrow$, $\theta_s \rightarrow$

θ_1		0	30	60	90	120	150	180	210	240	270	300	330
0	(a)	0	.40	.74	.66	.49	.66	.90	1.40	1.49	1.08	.45	-.17
	(b)		.37	.71	.62	.53	.72	.92	1.48	1.51	1.11	.49	-.13
	(c)		.03	.03	.04	-.04	-.06	-.02	-.08	-.02	-.03	-.04	-.04
30	(a)	0	.03	.23	.31			.83			.17		
	(b)		.04	.23									
	(c)		-.01	.00									
60	(a)	0	-.08	.27	.82			.43			.07		
	(b)		-.11	.25									
	(c)		.03	.02									
90	(a)	0			1.49			-.08			.51		
	(b)				1.52			-.05			.55		
	(c)				-.03			-.03			-.04		
180	(a)	0			-.34			-1.56			-.20		
	(b)				-.35			-1.51			-.22		
	(c)				.01			-.05			.02		
270	(a)	0			.04			-.33			1.44		
	(b)				.06			-.41			1.39		
	(c)				-.02			.08			.05		
$\delta_I(\theta_s)$		-.03	.00	.00	.10	.07	.03	-.16	-.10	.00	.07	.13	.07

6. Phase Calibration Corrections: $\epsilon_I(\theta)$, $\epsilon_{II}(\theta)$

θ	$\epsilon_I(\theta)$	$\epsilon_I(\theta+360^o)^{(4)}$	$\epsilon_{II}(\theta)$	$\epsilon_{II}(\theta+360^o)^{(4)}$
0	0^o	$-.03^o$	0^o	$.03^o$
30(2)	.32	.32	-.05	.02
60(2)	.31	.31	-.40	-.40
90(1)	.15	.25	-.47	-.47
120(3)	.16	.23	-.37	-.40
150(3)	.66	.69	-.06	-.13
180(1)	1.20	1.04	.28	.15
210(3)	1.43	1.33	-.05	-.12
240(3)	1.02	1.02	-.49	-.39
270(1)	.38	.45	-.73	-.60
300(3)	-.24	-.11	-.73	-.66
330(3)	-.35	-.28	-.22	-.22

(1) $\epsilon_I(\theta)$, $\epsilon_{II}(\theta)$ computed from Equations (18), (21)

(2) $\epsilon_I(\theta)$, $\epsilon_{II}(\theta)$ computed from Equations (19), (22)

(3) $\epsilon_I(\theta)$, $\epsilon_{II}(\theta)$ computed from Equations (20), (23)

(4) $\epsilon_I(\theta + 360)$, $\epsilon_{II}(\theta + 360)$ computed from Equations (5), (9)